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THE KINETIC MODEL OF A CARRIER PHASE IN A HETEROGENEOUS MEDIUM

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To describe a rarefied gas suspension we normally make use of a system of Boltzmann equations (BE), written for each of the components (see the review in [1]). Regimes which allow for such a description are examined in [2] and these are characterized by a system of inequalities ($i, j = 1, 2$): $r_i \ll d_i$, $\max\limits_i r_i \leq \ell_j$ ($j \neq i$), where r_i denotes the radii of the mixture components, $d_i = n_i^{-1/3}$, n_i, ℓ_i is the numerical density and the mean free path of the i-th component [$\ell_i \sim (r_i^2 n_i)^{-1}$]. In [3-6] we find a method for the solution of this system through various Enskog–Chapman (ECh) modifications.

Among the advantages in studying systems described by BE is the utilization of kinetic models. With this approach consideration of the complex physicochemical processes occurring at the surface of a particle is reduced to the calculation of the appropriate coefficients of the model (expressed in terms of the exchange coefficients), as well as the transition to the macroscopic description (including the derivation of expressions for the transfer coefficients) are realized considerably more simply than in the solution of the complete BE by the ECh method. The different versions of these kinetic models for mixtures were studied in [2, 7-9]. In the present study we examine the question of the construction of a kinetic model for the light component and its analysis within the framework of the ECh method, given an arbitrary function for the distribution of the heavy component.

The following BE system serves as the basis of our study:

$$df_1/dt_1 = J_{11}(f_1, f_1) + J_{12}(f_1, f_2),\ df_2/dt_1 = J_{22}(f_2, f_2) + J_{21}(f_2, f_1),$$

where $d/dt_i = \partial/\partial t + \mathbf{v}_i \cdot \partial/\partial \mathbf{r}$.

Let us examine a heterogeneous mixture characterized by substantial differences in mass and characteristic radii of the components $\varepsilon^2 = m_1/m_2 \ll 1$, $r_1 \ll r_2$. In this case, the reference mass $\mu_{12} \sim m_1$, and in evaluating the scattering cross section it is possible to assume that $\sigma_{11} \sim r_1^2$, $\sigma_{12} \sim r_2^2$, $\sigma_{22} \sim 4\sigma_{12}$.

For the collision terms J_{ij} we will use Boltzmann-type collision integrals written in symmetrized form:

$$J_{ij} = \frac{(m_1 m_2)^3}{\mu_{ij}} \int d\mathbf{v}_j\, d\mathbf{v}_i'\, d\mathbf{v}_j'\, \delta_\mathbf{p} \delta_E \sigma_{ij}^d \left(f_i' f_j' - f_i f_j\right), \tag{1}$$

where $\delta_\mathbf{p}$ and δ_E are the delta-functions of the conservation of momentum and of the kinetic energy of the colliding pair; the primes denote that a given quantity belongs to the characteristics of state after collision; σ^d is the differential scattering cross section whose analytical approximations for elastic collisions have been studied in detail in [10]. In particular, in order to calculate the cross section of the collision between the light component and a heavy component, as well as within the heavy component, it was proposed in [11] to describe the corresponding interactions by means of the Kihara potential.

Construction of the kinetic model [i.e., a finite-multiple approximation of integral (1)] involves two stages: the finding of the quasisteady distributions of f_i^0; the expan-

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sion of the distribution function and the collision integral over sets of the functions ψ_α.

The quasisteady distributions of f_i^0 are determined by a set of slow variables $\Gamma_{i\gamma} = (f_i, \chi_{i\gamma})$, where χ_γ represents the approximate collision invariance (CI) [12]:

$$n_i^{-1}(\chi_{i\gamma}, J_i) \leqslant O(k_i), \tag{2}$$

$J_i = \sum_j J_{ij}$, $(\varphi, \psi) = \int \varphi\psi\, d\mathbf{v}$, k_i is the Knudsen number of the i-th component; it is understood that χ has been made dimensionless in appropriate fashion.

In the construction of the kinetic model for the light component we will dwell on the situation in which it can be described in the terminology of ordinary slow (hydrodynamic) variables n_1, $\mathbf{u}_1$, T_1 (density, velocity, and temperature). As the function near which we will linearize the light-component distribution function we will take the Maxwell–Boltzmann distribution ($\varphi_1 \ll 1$)

$$f_1 = f_{10}(1 + \varphi_1), \quad f_{10}(\mathbf{v}) = n_1\left(\frac{m_1}{2\pi k T_1}\right)^{3/2} \exp\left(-m_1\frac{(\mathbf{v} - \mathbf{u}_1)^2}{2kT_1}\right). \tag{3}$$

From the conditions imposed on the distribution function and from the properties of f_{10} we obtain the relationship

$$\int f_{10}\varphi_1\{1, \mathbf{v}, (1/2)(\mathbf{v} - \mathbf{u}_1)^2\} d\mathbf{v} = 0. \tag{4}$$

Before we turn to the direct notation of the model kinetic equation, let us take note of the fact that in its solution by the ECh method the function φ_1 is expanded into a series of the form

$$\varphi_1 = \sum_{q=0} k_1^q \varphi_{1(q)}. \tag{5}$$

The question as to the term from which this expansion begins depends on the solution of the equation for the quasisteady distribution

$$J_{11}(f_1^0) + J_{12}(f_1^0, f_2) = o(k_1). \tag{6}$$

If the intercomponent interaction determined by the term J_{12} in (6) leads to a marked deviation of f_1^0 from f_{10} in the sense that $1 \gg \varphi_{1(0)} = (f_1^0 - f_{10})/f_{10} \gg k_1$ [with $\varphi_{1(0)}$ satisfying relationships (4)], then series (5) begins from the term q = 0, and in the hydrodynamic equations changes will begin in the zeroth order of the Knudsen number. However, if the intercomponent interaction introduces small perturbations, the deviation from the equilibrium function f_{10} will be small [$\leq O(k_1)$] and the correction factors in the hydrodynamic equations will appear only in the Navier–Stokes approximations.

Let us now turn to the construction of the model. The function φ_1 from (3) will be expanded over a base system formed by a combination of Sonin polynomials and irreducible tensors [13]:

$$\varphi_1 = \sum_{lm} a_{l,(\mu)_l,m}\psi_{lm}Y_{l,(\mu)_l}.$$

The expansion coefficients a are tensor functions of order ℓ, depending on the spatial and time variables. Summation over ℓ symbolizes the summation over the ℓ subscripts $(\mu)_\ell$. To simplify the notation we will subsequently sometimes indicate only the order of the tensor; $Y_{\ell,(\mu)_\ell}$ denotes the irreducible tensors of order ℓ, $\psi_{\ell m}(c^2)$ represents the total system of orthogonal polynomials:

$$\psi_{lm} = \left[\frac{\sqrt{\pi}}{2}\frac{m!}{\Gamma(l + m + 3/2)}\right]^{1/2} S^m_{l+1/2}(c_1^2), \quad \mathbf{c} = \sqrt{\frac{m_1}{2kT_1}}(\mathbf{v}_1 - \mathbf{u}_1),$$

$S_{\ell+1/2}^m(c^2)$ denotes the Sonin–Laguerre polynomials. The explicit form of some of the first functions of Y and ψ can be found, for example, in [13].

Relationship (4) generates certain relationships between the coefficient a and the hydrodynamic quantities:

$$
\begin{gathered}
a_{00} = a_{1,\alpha,0} = a_{01} = 0, \\
\Pi_{1\alpha\beta} = P_{1\alpha\beta} - p_1\delta_{\alpha\beta} = \sqrt{\frac{4}{15}}\, n_1 k T_1 \mathring{a}_{2,\alpha\beta,0}, \quad p_1 = n_1 k T_1, \\
q_{1\alpha} = -n_1 k T_1 \sqrt{\frac{5kT_1}{6m_1}}\, a_{1,\alpha,1} \\
\left(\mathring{a}_{2,\alpha\beta,0} \equiv \frac{1}{2}(a_{2,\alpha\beta,0} + a_{2,\beta\alpha,0}) - \frac{1}{3} a_{2,\gamma\gamma,0}\delta_{\alpha\beta}\right).
\end{gathered} \tag{7}
$$

The Greek subscripts here denote the Cartesian coordinates of the stress tensor $\mathbf{P}$ and the heat-flow vector $\mathbf{q}$, determined from the relationships

$$
\mathbf{P}_1 = \int m_1 (\mathbf{v} - \mathbf{u}_1)(\mathbf{v} - \mathbf{u}_1) f_1 d\mathbf{v}, \quad \mathbf{q}_1 = \int \frac{m_1}{2} (\mathbf{v} - \mathbf{u}_1)^2 (\mathbf{v} - \mathbf{u}_1) f_1 d\mathbf{v}. \tag{8}
$$

The integral for the light component, linearized with the aid of (3), can be represented in the form of two terms:

$$
\begin{gathered}
L = L^0 + L^1(\varphi_1), \; L^0 = J_{12}(f_{10}, f_2), \\
L^1(\varphi_1) = \frac{m_1^6}{\mu_{11}^2} \int d\tilde{\mathbf{v}}_1\, d\mathbf{v}_1'\, d\tilde{\mathbf{v}}_1' \delta_{\mathbf{p}} \delta_E \sigma_{11}^d f_{10}(\mathbf{v}_1) f_{10}(\tilde{\mathbf{v}}_1) [[\varphi_1]] + \\
+ \frac{(m_1 m_2)^3}{\mu_{12}} \int d\mathbf{v}_2\, d\mathbf{v}_1'\, d\mathbf{v}_2' \delta_{\mathbf{p}} \delta_E \sigma_{12}^d \left(f_{10}' f_2' \varphi_1' - f_{10} f_2 \varphi_1\right), \\
[[\varphi]] \equiv \varphi' + \tilde{\varphi}' - \varphi - \tilde{\varphi}.
\end{gathered} \tag{9}
$$

Expanding L over the same basis system of functions as φ, we obtain

$$
\frac{d}{dt_1}\varphi_1 + (1 + \varphi_1)\frac{d}{dt_1}\ln f_{10} = \sum_{kl} A_{kl}\psi_{kl} Y_k + \sum_{klmn} B_{mn}^{kl} a_{mn} \psi_{kl} Y_k, \tag{10}
$$

where the expansion coefficients are determined in terms of L^0 and L^1:

$$
\begin{gathered}
A_{kl} = \frac{1}{n_1 Q_k}(L^0, \psi_{kl} Y_k), \quad B_{mn}^{kl} = \frac{1}{n_1 Q_k}(L^1(\psi_{mn} Y_m), \quad \psi_{kl} Y_k), \\
Q_k = \frac{1}{4\pi} \int_{[4\pi]} Y_k(\mathbf{c}^0) Y_k(\mathbf{c}^0)\, d\mathbf{c}^0.
\end{gathered} \tag{11}
$$

In the expression for Q_k it is understood that we are dealing with a scalar tensor product, and the integration is performed over the solid angle corresponding to the vector $\mathbf{c}^0 = \mathbf{c}/c$. The same scalar product is used in (11) as in (2). It is easy to obtain $A_{00} = 0$. The coefficients B are presented in the form of two terms $B = B^0 + B'$, corresponding to the first and second terms in L^1 from (9). The properties of B^0 have been thoroughly studied (see, for example, [13]) and we will not dwell in detail on these here, but we will take note only of the fact that

$$
B_{mn}^{0kl} = B_{kl}^{0mn}, \quad B_{00}^{0kl} = B_{10}^{0kl} = B_{01}^{0kl} = 0. \tag{12}
$$

For a final formulation of the model it is necessary to select a method to "interrupt" the infinite sums in (10). Proceeding in the spirit of [13], we will carry out the substitution: with $|k + 2\ell| > N$, $|m + 2n| > N$ we assume that $A_{k\ell} = 0$ and $B_{mn}{}^{k\ell} \to -\nu_N\delta_{km}\delta_{\ell n}$, where the first Kronecker delta represents the dimensional tensor $2k = 2m$. For ν_N it is natural to use the usual approximation $\nu_N = -B_{N,0}{}^{0N,0}$.

The proposed procedure allows us to formulate a model of order N in the form

$$
D\varphi_1 = \sum_{|k+2l| \leqslant N} A_{kl}\psi_{kl} Y_k + \sum_{|k+2l| \leqslant N} \left(B_{mn}^{kl} + \nu_N \delta_{km}\delta_{ln}\right) a_{mn}\psi_{kl} - \nu_N \varphi_1
$$

[D is the operator in the left-hand side of (10)]. We note that in contrast to the isotropic case, in the N-th-order model derived here it is no longer possible to eliminate the N-order irreducible tensor.

The simplest model which takes into consideration the presence of the proposed five approximate collisions invariants is the second-order model. To eliminate the cumbersome calculations which arise in higher-order models, we will demonstrate the fundamental features introduced by the impurity phase on the example of this model. Using the properties of the coefficients A and B [see (12) and the text above], we obtain (understood here is summation over repeating subscripts)

$$\begin{aligned}D\varphi_1 = &\left(A_{01} + \sqrt{\frac{4}{15}}\frac{\Pi_{\xi\eta}}{p_1} B'^{01}_{2,\xi\eta,0}\right)\psi_{01}Y_0 + \\ &+\left(A_{1,\alpha,0} + \sqrt{\frac{4}{15}}\frac{\Pi_{\xi\eta}}{p_1} B'^{1,\alpha,0}_{2,\xi\eta,0}\right)\psi_{10}Y_{1,\alpha} + \\ &+\left(A_{2,\alpha\beta,0} + \sqrt{\frac{4}{15}}\frac{\Pi_{\xi\eta}}{p_1} B'^{2,\alpha\beta,0}_{2,\xi\eta,0}\right)\psi_{20}Y_{2,\alpha\beta} - \nu_2\varphi_1.\end{aligned} \tag{13}$$

Here we used $\mathrm{Sp}_{\xi\eta}B'^{2,\alpha\beta,0}_{2,\xi\eta,0} = 0$ and the relationship of $a_{2,\xi\eta,0}$ to the stress tensor from (7). Let us note that N-order model, just as in the single-component case, does not contain $B_N{}^0$, but it contains only the values of $B_N{}'$. Let us ascertain the physical sense of the coefficients A. We will examine the quantities

$$F_{12\alpha} = \int m_1 v_{1\alpha} J_{12}\, d\mathbf{v}_1, \quad M_{12\alpha\beta} = \int m_1 (v_1 - u_1)_\alpha (v_1 - u_1)_\beta J_{12}\, d\mathbf{v}_1,$$
$$Q_{12} = \frac{1}{2}\mathrm{Sp}\, M_{12} = \int \frac{m_1}{2} v_1^2 J_{12}\, d\mathbf{v}_1 - \mathbf{u}_1\cdot\mathbf{F}_{12},$$

characterizing the transmitted momentum, stress, and energy. Using the explicit form of the functions ψ and Y, we obtain

$$A_{1,\alpha,0} = \sqrt{\frac{2}{3}}\frac{F^0_{12,\alpha}}{n_1\sqrt{2m_1 kT_1}}, \quad A_{01} = -\sqrt{\frac{2}{3}}\frac{Q^0_{12}}{n_1 kT_1}.$$

Analogously, A_{20} is expressed in terms of M^0. The superscript 0 with F, Q, and M indicates that these quantities are calculated with the equilibrium distribution function f_{10}.

By means of the usual procedure we can make the transition from (13) to the transfer equation:

$$\begin{aligned}&\frac{\partial n_1}{\partial t} + \frac{\partial n_1\mathbf{u}_1}{\partial \mathbf{r}} = 0, \quad \frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_1\cdot\frac{\partial \mathbf{u}_1}{\partial \mathbf{r}} = \mathbf{F}_1 - \frac{1}{\rho_1}\frac{\partial}{\partial \mathbf{r}}\mathbf{P}_1, \\ &\frac{\partial kT_1}{\partial t} + \mathbf{u}_1\cdot\frac{\partial kT_1}{\partial \mathbf{r}} = -\frac{2}{3n_1}\left(\frac{\partial \mathbf{q}_1}{\partial \mathbf{r}} + \mathbf{P}_1 : \frac{\partial \mathbf{u}_1}{\partial \mathbf{r}}\right) - \Xi_1,\end{aligned} \tag{14}$$

where $\mathbf{P}_1$ and $\mathbf{q}_1$ were introduced into (8), while $\mathbf{F}_1$ and Ξ_1 for the second-order model have the form

$$\begin{aligned}F_{1\gamma} &= \sqrt{\frac{kT_1}{3m_1}}\left(A_{1,\gamma,0} + \sqrt{\frac{4}{15}}\frac{\Pi_{1\alpha\beta}}{p_1} B'^{1,\gamma,0}_{2,\alpha\beta,0}\right), \\ \Xi_1 &= kT_1\sqrt{\frac{2}{3}}\left(A_{01} + \sqrt{\frac{4}{15}}\frac{\Pi_{1\alpha\beta}}{p_1} B'^{01}_{2,\alpha\beta,0}\right).\end{aligned} \tag{15}$$

It is essential that we solve Eq. (13) in order to close system (14). Let us first take a look at the situation in which the intercomponent interaction changes the equilibrium distribution in the zeroth order of the Knudsen number. Such a distribution is given by relationship (6), which for our model indicates the vanishing of the right-hand side of (13) as $\varphi_1 \to \varphi_{1(0)}$. The latter relationship is easily expanded for $\varphi_{1(0)}$. Taking into consideration that $\varphi_{1(0)}$ must also satisfy relationships (4), we obtain

$$\varphi_{1(0)} = \nu_2^{-1}\left(\mathring{A}_{2,\alpha\beta,0} + \sqrt{\frac{4}{15}}\frac{\Pi_{\xi\eta}}{p_1} B'^{2,\alpha\beta,0}_{2,\xi\eta,0}\right)\psi_{20}Y_{2,\alpha\beta}. \tag{16}$$

Using this result to calculate the stress tensor according to (8), we arrive at the matrix equation

$$\Pi_{ij} = \sqrt{\frac{4}{15}}\,\nu_2^{-1} p_1 \mathring{A}_{2,ij,0} + \Delta_{ij}^{\xi\eta}\Pi_{\xi\eta},$$
$$p_1 = n_1 k T_1, \quad \Delta_{ij}^{\xi\eta} = \frac{4}{15}\,\nu_2^{-1} B'^{2,ij,0}_{2,\xi\eta,0}. \tag{17}$$

Using the properties of $A_{2,0}$ and $B_{20}'^{20}$, it is easy to demonstrate that $\mathrm{Sp}\,\Pi = 0$ and $\Pi_{\alpha\beta} = \Pi_{\beta\alpha}$.

The absence of a flow of heat: $\mathbf{q} = 0$ follows out of (16). Here we see the "defect" of the second-order model. For the third-order model in zeroth approximation of k_1 we obtain a nonzero flow of heat, one that can be ascribed to the absence of equilibrium in the two-component medium.

In order to analyze the numerous features contained in expressions (15) and (17), let us examine the extensive flow regime corresponding to the case $\max\{k_2, k_2/\alpha_{21}\} \gg 1$, where k_2 is the Knudsen number for the heavy component, and α_{21} is the parameter by means of which the collision integral J_{21} is made dimensionless, and at the same time characterizing the interphase interaction. In this regime f_2 is the slow variable (the so-called collision-free regime). Then, as the quasisteady distribution of the impurity phase we can choose f_{2in} which represents the initial distribution. Such a distribution is frequently Maxwellian in nature with the parameters $\mathbf{u}_{20}$, T_{20}. We note that these quantities are not flow variables, but only parameters of the function f_{2in}, i.e., the relaxation, generally speaking, occurs with a slow change in the very form of the function f_2, and not only of its parameters. A similar situation exists, for example, in the injection of a Maxwell bundle of particles into a stream of gas or in the passage of a shock wave through a cloud of dust, when each of the phases ahead of the shock-wave front is in equilibrium with the total hydrodynamic velocity and the overall temperature.

Such a choice for the quasisteady distribution makes it possible directly to calculate the coefficients A and B. The expressions for these quantities are even more simplified if we make use of the smallness of the parameter ε. In (11) we turn to the new integration variables by means of the substitutions

$$\mathbf{v}_1 = \mathbf{V} + \frac{1}{1+\varepsilon^2}\mathbf{G}, \quad \mathbf{v}_2 = \mathbf{V} - \frac{\varepsilon^2}{1+\varepsilon^2}\mathbf{G}, \quad \mathbf{g} = \sqrt{\frac{m_1}{2kT_1}}\,\mathbf{G}$$

and the analogous substitution for the variables identified with primes. In view of the assumptions made above, the functions $f_1(\mathbf{v}_1)$ and $f_2(\mathbf{v}_2)$ contained within the coefficients of the model differ little from the Maxwell functions, and we can therefore assume that they have a characteristic form with the maximum at $\mathbf{v}_i = \mathbf{u}_i = n_i^{-1}\int \mathbf{v}_i f_i\, d\mathbf{v}_i$ (the average velocity) and with the width of the maximum $\sqrt{m_i/2kT_i}$. As a consequence of normalization and because of the relationship $\varepsilon \ll 1$ the function f_2 exhibits a sharp peak, and within the scope of the asymptotic Laplace method the remaining functions may be regarded as rather smooth and that they can be calculated at the point $\mathbf{v}_2 = \mathbf{u}_2$. The remaining integral is calculated in trivial fashion, taking into consideration that $\int f_2\, d\mathbf{v}_2 = n_2$. In this expression we will neglect the terms $\sim\varepsilon^2$, and this will give us

$$B_{ij}'^{mn} = \frac{n_2}{\pi Q_m}\sqrt{\frac{2kT_1}{\pi m_1}}\int g^3\, dg\, d\mathbf{n}\, d\mathbf{n}'\sigma^d(g, \mathbf{n}\cdot\mathbf{n}')\exp(-(\mathbf{g}-\mathbf{w})^2)\times$$
$$\times\,\psi_{ij}((\mathbf{g}-\mathbf{w})^2)\,Y_i(\mathbf{g}-\mathbf{w})\,[\psi_{mn}((\mathbf{g}'-\mathbf{w})^2)\,Y_m(\mathbf{g}'-\mathbf{w}) - \psi_{mn}((\mathbf{g}-\mathbf{w})^2)\,Y_m(\mathbf{g}-\mathbf{w})]; \tag{18}$$
$$Q_0 = Q_1 = 1,\ Q_2 = 2/3,\ Q_3 = 4/25,\ \mathbf{w} = \sqrt{m_1/2kT_1}(\mathbf{u}_1 - \mathbf{u}_{20}).$$

It is easy to find the expressions for A if we use the relationship $A_{mn} = B_{00}'^{mn}$.

Let us now turn to the calculation of the stress tensor from (17) and the characteristics of the interphase interaction (15). In approximation of the small anisotropy ($\|\Delta\| \ll 1$) in the initial stage of perturbation theory we will obtain $(I_{\alpha\beta}^{\varphi\psi} = \delta_{\alpha\varphi}\delta_{\beta\psi})$

$$\Pi_{\alpha\beta} = \sqrt{\frac{4}{15}}\,\nu_2^{-1} p_1 (I_{\alpha\beta}^{\varphi\psi} - \Delta_{\alpha\beta}^{\varphi\psi})^{-1}\mathring{A}_{2,\varphi\psi,0} \approx \sqrt{\frac{4}{15}}\,\nu_2^{-1} p_1 (A_{2,\alpha\beta,0} + \Delta_{\alpha\beta}^{\varphi\psi} A_{2,\varphi\psi,0}). \tag{19}$$

For a cross section of the interaction we will take the model of solid spheres in zeroth approximation for Δ and we will find (the expressions for the coefficients A and B can be found in the Appendix)

$$P_{\alpha\beta} = p_1\delta_{\alpha\beta} + \frac{\sqrt{6}}{5}\nu_2^{-1}\Phi(w)\,w^2W_{\alpha\beta},\quad W_{\alpha\beta} = w^{-2}\left(w_\alpha^0 w_\beta\right),$$
$$\Phi(w) = 4n_2\sigma_{12}\sqrt{\frac{kT_1}{3\pi m_1}}\left[1 + \frac{2}{w}\int\limits_0^w dq\,\mathrm{e}^{-q^2}\left(w^2q^2 - wq^3 + \frac{1}{3}q^4\right)\right]. \tag{20}$$

Thus, the presence of the second component leads to the appearance of a substantially anisotropic term in the stress tensor. In the approximation with which we are dealing here, additional "pressure" arises only in the direction of the relative velocity. When we take into consideration the second term in (19) additional contributions to the pressure arise in all directions. We are not going to write out the corresponding expressions here, because they are cumbersome; however, these can easily be reproduced by using the expression from the Appendix identified (A.2). The correction factors to the stress tensor, derived earlier, are naturally referred to as the relaxation pressure, since they disappear as the system reaches equilibrium, a point at which the velocities are equalized ($\mathbf{w} = 0$).

For the force of the interphase interaction within the scope of these assumptions we have

$$F_{1\gamma} = \sqrt{\frac{kT_1}{3m_1}}\left\{\Phi(w)\,w_\gamma + \frac{2}{5}\sqrt{\frac{2}{5}}\nu_2^{-1}\Phi(w)\,w^2\left[a_1W_{\alpha\beta}^2\frac{w_\gamma}{w} + a_2\left(W_{\gamma\beta}\frac{w_\beta}{w} + W_{\alpha\beta}\frac{w_\alpha}{w}\right)\right]\right\},$$
$$a_i = \frac{4}{3}n_2\sigma_{12}\sqrt{\frac{kT_1}{5\pi m_1}}\int\limits_0^\infty dq\,q^4\mathrm{e}^{-q^2}\int\limits_{-1}^{1}dx\,\sqrt{q^2 + w^2 + 2qwx}\,\xi_i, \tag{21}$$
$$\xi_1 = q(5x^3 - 3x) + w(3x^2 - 1),\quad \xi_2 = qx(1 - x^2).$$

As we can see from (21), in addition to the component along the relative velocity, the intercomponent force $\mathbf{F}_1$ also has components in the remaining directions, which may be attributed to the significant perturbation of the distribution function for the light component.

The interphase exchange of energy is determined by means of the expression

$$\Xi_1 = -2\sqrt{\frac{2}{3}}kT_1\Phi(w)\,w\left(w + (a_1 + 2a_2)\frac{2}{5}\sqrt{\frac{2}{5}}\nu_2^{-1}W_{\alpha\beta}^2\right) \tag{22}$$

[$a_{1,2}$ are the same as in (21)].

Relationships (20)-(22) are closed in the zeroth approximation of the Knudsen number in the transfer equation (14) when the impurity component exerts considerable influence. Extremely cumbersome expressions appear in the first order with respect to k_1, and we will not dwell on these here. In the relaxation process the relative velocity w diminishes and the perturbation of the distribution function becomes small, i.e., series (5) begins from the term with $q = 1$ ($\varphi_{1(0)} = 0$). In this case the scheme for the solution of Eq. (13) changes, since in the zeroth approximation of k_1 we have $f_1 = f_{10}$. Consequently, with the closure of the transport equation (14) as $k_1 \to 0$ we have $P_{1\alpha\beta} = p_1\delta_{\alpha\beta}$, which means $\Pi_{\alpha\beta} = 0$. The effects of anisotropy are small and do not become apparent in the approximation under consideration.

In the first approximation of the parameter k_1 from (13) we have

$$\varphi_1 = -\nu_2^{-1}\left[\left(c^2 - \frac{5}{2}\right)(\mathbf{v}_1 - \mathbf{u}_1)\cdot\nabla kT_1 + 2\,(c^0c)_{\alpha\beta}\colon\left(\mathring{U}_{1\alpha\beta} - \right.\right.$$
$$\left.\left. - \frac{2}{15}\frac{\Pi_{1\xi\eta}}{p_1}B'^{2,\alpha\beta,0}_{2,\xi\eta,0}\right)\right],\quad \mathring{U}_{1\alpha\beta} = (\nabla^0u_1)_{\alpha\beta} - \sqrt{\frac{1}{15}}\mathring{A}_{2,\alpha\beta,0}. \tag{23}$$

Substitution of (23) into expression (8) for $\mathbf{P}$ leads to the matrix equation

$$\Pi_{\alpha\beta} = \Gamma_{\alpha\beta} + \Delta_{\alpha\beta}^{\varphi\psi}\Pi_{\varphi\psi},\quad \mathbf{\Gamma} = -2\mu_1\mathring{\mathbf{U}}_1,\quad \mu_1 = n_1kT_1\nu_2^{-1} \tag{24}$$

[Δ is the same as in (17)]. Using the properties of $\boldsymbol{\Gamma}$ and $B_{20}{}'^{20}$, it is easy to show that $\mathrm{Sp}\,\boldsymbol{\Pi} = 0$ and $\Pi_{\alpha\beta} = \Pi_{\beta\alpha}$.

An analogous procedure for the heat-flow vector leads to $\mathbf{q} = -\lambda_1 \nabla \ln kT_1$, $\lambda_1 = (15/4)\cdot(k/m_1)\mu_1$. Let us note that in the third-order model for $\mathbf{q}$ we now obtain a matrix equation such as (24), and in the corresponding analog we have a term additional to $\nabla \ln kT_1$ (the analog A_{20}). In the case of low anisotropy, in analogy with (19), we obtain

$$P_{\alpha\beta} = p\delta_{\alpha\beta} + \Gamma_{\alpha\beta} + \Delta_{\alpha\beta}^{\varphi\psi}\Gamma_{\varphi\psi}. \tag{25}$$

The results obtained here indicate the non-Newtonian nature of the flow and the anisotropy of the viscosity coefficient. Let us note that the anisotropy of the stress tensor (both in the case of limited and pronounced effect on the part of the impurity component) is essentially a reflection of the fact that the presence of the impurity, not in equilibrium with the carrier component and exerting a directed action on the latter, causes the carrier phase to be a nonclosed nonisotropic system.

In conclusion we will write out the structure of the stress tensor determined from relationship (25) by using approximation (18) for the coefficients, and by using the expressions derived in the Appendix:

$$\begin{gathered}P_{1\alpha\beta} = (p_1 + p_1^*)\,\delta_{\alpha\beta} - 2\mu_{1\alpha\beta}^{*mn}\mathring{U}_{1mn} + 2\mu_1\tilde{b}_3(\mathring{U}_{1\beta n}w_n w_\alpha + \mathring{U}_{1\alpha n}w_n w_\beta)/w^2,\\ p_1^* = -\tilde{b}_0\Gamma_{mn}W_{mn}, \quad \mu_{1\alpha\beta}^{*mn} = \mu_1\left[(1-\tilde{b}_1)I_{\alpha\beta}^{mn} - \tilde{b}_2 W_{mn}\frac{w_\alpha w_\beta}{w^2}\right].\end{gathered} \tag{26}$$

Calculation of the coefficient B' with the transport scattering cross sections dependent on their argument yields the following values for the parameters:

$$\tilde{b}_i = \frac{4}{15}\nu_2^{-1}b_i, \quad b_i = \frac{n_2\sigma_{12}}{10}\sqrt{\frac{2kT_1}{m_1}}\int_0^\infty dq q^4 e^{-q^2}\int_{-1}^{1} dx\,\sqrt{q^2+w^2+2qwx}\,\eta_i,$$

$$\eta_0 = \frac{1}{3}\,[q^2(6x^2-15x^4+1)+4w^2(3x^2-1)], \quad \eta_1 = 2q^2(x^2-1)^2,$$

$$\eta_2 = q^2(35x^4-30x^2+3)-4w^2(3x^2-1),\ \eta_3 = 2q^2(6x^2-5x^4-1).$$

The p_1^* and μ_1^* contained in the expression for the stress tensor have the sense of relaxation pressure and the tensor of effective viscosity. Thus, the rheology of the two-component medium differs substantially from the Newtonian both as a result of the fact that the stress tensor is a function of velocity and as a result of the anisotropy of the viscosity coefficient. Moreover, an additional term arises in the stress tensor [the last term in (26)], which exhibits a totally different structure: the stresses are proportional not only to the elements of the strain-rate tensor but also to the dyads made up of the relative velocities.

We should take note of the fact that the observed structure of the hydrodynamics equations for the carrier component may be significant in the study of the flow stability of two-component mixtures and the propagation of sound in such media.

APPENDIX

On the basis of expressions (18), for the coefficient B' of the model and the indicated relationship between the coefficients A and B' we obtain a representation for these in the form of quadratures containing σ_d and σ_v, i.e., the cross section of the diffusion and of the viscosity [for the solid-sphere model $(3/2)\sigma_v = \sigma_d = \sigma$]:

$$\nu_2 = \frac{64n_1}{3}\sqrt{\frac{kT_1}{15\pi m_1}}\int x^2 e^{-x}\left(\sigma_{11}+\frac{3}{2}\sigma_{v11}\right)dx, \quad A_{1,\alpha,0} = -\Phi(w)\,w_\alpha,$$

$$A_{01} = 2w_\alpha A_{1,\alpha,0} = -2\Phi(w)\,w^2, \quad A_{2,\alpha\beta,0} = \frac{3}{\sqrt{10}}\Phi(w)\,w^2 W_{\alpha\beta}$$

[the function $\Phi(w)$ and the tensor W were introduced in (20)],

$$B_{2,\alpha\beta,0}^{\prime 1,\gamma,0} = -a_1\frac{w_\alpha w_\beta w_\gamma}{w^3} - a_2\left(\delta_{\alpha\gamma}\frac{w_\beta}{w}+\delta_{\beta\gamma}\frac{w_\alpha}{w}\right) - \delta_{\alpha\beta}\frac{w_\gamma}{w}\left[a_3 w + a_2 - \frac{a_4+a_5 w}{3}\right],$$

$$a_i = \frac{4n_2}{3}\sqrt{\frac{kT_1}{5\pi m_1}}\int dq\, q^4 e^{-q^2}\int_{-1}^{1} dx\sqrt{q^2+w^2+2qwx}\,\sigma_{d12}\xi_i,$$

$$\xi_1 = q(5x^3-3x)+w(3x^2-1),\ \xi_2 = qx(1-x^2),$$
$$\xi_3 = 1-x^2,\ \xi_4 = q,\ \xi_5 = 1$$

[the coefficients $a_{1,2}$, calculated in the solid-sphere model, are shown in (21)],

$$B'^{01}_{2,\alpha\beta,0} = 2w_\gamma B'^{1,\gamma,0}_{2,\alpha\beta,0}; \tag{A.1}$$

$$B'^{2,mn,0}_{2,ij,0} = -b_1\delta_{ij}\delta_{mn} - b_2W_{ij}\delta_{mn} - b_3\delta_{ij}W_{mn} - b_4W_{ij}W_{mn} - \tag{A.2}$$
$$- b_5(\delta_{mi}\delta_{ni}+\delta_{mj}\delta_{ni}) - b_6(W_{jn}\delta_{im}+W_{in}\delta_{jm}+W_{jm}\delta_{in}+W_{im}\delta_{jn}),$$

$$b_i = \frac{n_2}{10}\sqrt{\frac{2kT_1}{\pi m_1}}\int_0^\infty dq\, q^4 e^{-q^2}\int_{-1}^{1}\sqrt{q^2+w^2+2qwx}\,\zeta_i,$$

$$\zeta_1 = \frac{q}{3}\left[\sigma_{v12}q(7x^4-6x^2-1)+\left(\frac{3}{2}\sigma_{v12}-\sigma_{d12}\right)\frac{w}{3}(-19x^3+3x+2)\right],$$

$$\zeta_2 = 2q\left[\sigma_{v12}q(5x^4-6x^2+1)+\left(\frac{3}{2}\sigma_{v12}-\sigma_{d12}\right)\frac{2wx}{3}(7x^2-5)\right],$$

$$\zeta_3 = 2q\left[\sigma_{v12}q(5x^4-6x^2+1)+\left(\sigma_{v12}-\frac{2}{3}\sigma_{d12}\right)\frac{8wx}{3}(x^2-1)\right],$$

$$\zeta_4 = \frac{1}{2}\left[3\sigma_{v12}q^2(35x^4-30x^2+3)+16\left(\frac{3}{2}\sigma_{v12}-\sigma_{d12}\right)qwx(5x^2-3)+\right.$$
$$\left.+8\left(\frac{3}{2}\sigma_{v12}-2\sigma_{d12}\right)w^2(3x^2-1)\right],$$

$$\zeta_5 = \frac{1}{2}\left[\sigma_{v12}q^2(6x^2-7x^4+1)+\left(\frac{3}{2}\sigma_{v12}-\sigma_{d12}\right)\frac{16x}{3}(1-x^2)\right],$$

$$\zeta_6 = \frac{q}{2}\left[3\sigma_{v12}q(6x^2-5x^4-1)+8\left(\frac{3}{2}\sigma_{v12}-\sigma_{d12}\right)wx(1-x^2)\right].$$

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